Journal of Sound and Vibration (2000) **236**(3), 413–430
doi:10.1006/jsvi.2000.2996, available online at http://www.idealibrary.com on IDEAL®

REAL-TIME CONDITION MONITORING BY SIGNIFICANT AND NATURAL FREQUENCIES ANALYSIS OF VIBRATION SIGNAL WITH WAVELET FILTER AND AUTOCORRELATION ENHANCEMENT

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(*Received* 8 *October* 1999, *and in final form* 3 *February* 2000)

Vibration signals from machining processes contain very useful information and offer excellent possibilities for in-process condition monitoring. Significant and natural frequencies in vibration signals reflect the inherent property of the system. Any change in the condition will result in the change in these frequencies. The time–frequency localization of the wavelet transform makes it very useful in signal processing, so that it can be used in filter design. In this paper, the wavelet filter with very narrow frequency-band and autocorrelation enhancement is proposed as a means of monitoring the natural frequencies in real-time. With the efficient, reliable and flexible wavelet filters, any number of natural frequencies in either the time or the frequency domain can be observed at the same time. After filtering, the signal with a single central frequency is enhanced by an autocorrelation algorithm in the time domain. This can provide real-time monitoring based on the changes of significant and natural frequencies, corresponding to the change in conditions.

1. INTRODUCTION

The objective of any machining process is the efficient production of a part with specific shape with acceptable dimensional accuracy and surface quality. Deviation of the machine conditions from a prescribed plan may influence the final part quality and must be carefully examined by the machine operator. If the condition monitoring can be carried out automatically, the burden on machine operators and the risk of human error will be reduced. The need for such a monitoring system for efficient machine operator is clear.

Vibration signals carry a great deal of information about system condition. The vibration signatures of machines processes have been investigated as potential sources for an in-process monitoring tool [1]. Vibration monitoring presents a unique and attractive opportunity for condition monitoring. A simple approach (described in reference [2]) for estimating the machine's condition is to measure the overall vibration signal power and to compare this with the power measure when the machine is in a new and undamaged condition. Other approaches for estimating the machine's condition use a variety of features estimated from the vibration signal such as statistical comments or exploit frequency domain information through the use of power spectra. A periodic time-varying autoregressive model-based approach for the detection of bearing faults was presented in reference [2]. However, it is still a challenge to isolate and characterize only those specific signatures from the vibration signals which are relevant for the diagnosis of a particular condition.

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Significant and natural frequencies, which are determined via vibration (with free running) analysis and theoretical calculation, reflect the inherent property of the system. Any change in the system condition will result in the change in these frequencies. Therefore the frequencies can be used for system diagnosis. Recently, a method of checking the spindle assembly by making vibration measurements has been proposed [3]. It is concluded that some of the natural frequencies of machine tools that particularly involve the spindles are sensitive to preload. This in turn results in these frequencies being a suitable measure of the preload state of the bearings. It is possible to use these natural frequencies as a quality control for spindle assembly, both at the production stage and at any time for planned maintenance.

Defect detection method for rolling bearings through sensor signature analysis of vibration were investigated in reference [4], using high-frequency resonance technique (HFRT) and adaptive line enhance (ALE). The signal processing procedure is as follows.

(i) Band-pass filtering around resonance frequency.
(ii) Demodulation.
(iii) Low-pass filtering.
(iv) Re-sampling at a lower rate.
(v) Adaptive the enhancer (using recursive least mean squares algorithm).

The signal-to-noise ratio is increased through this method. The results also indicate that the frequency-based peak ratio is excellent for localized defect detection, and that a time domain method is more suited for non-localized defect detection. However, no suitable and simple approach is proposed for real-time signal processing and condition monitoring.

2. WAVELET TRANSFORM AND WAVELET FILTER

2.1. WAVELET TRANSFORM

Fourier analysis has been used effectively for many years. A disadvantage of Fourier analysis is that frequency information can only be extracted for the complete duration of a single $x(t)$. If at some point in the lifetime of $x(t)$, there is a local oscillation representing a particular feature, this will contribute to the calculated Fourier transform $X(\omega)$, but its location on the time axis will be lost. There is no way of knowing whether the value of $X(\omega)$ at a particular ω derives from frequencies present throughout the life of $x(t)$ or during just one or a few selected periods. This drawback is overcome in wavelet analysis, which provides an alternative way of breaking a signal down into its constituent parts. Wavelet analysis allows a general function of time to be decomposed into a series of basis functions, called wavelets, of different lengths and different positions along the time axis. A particular feature can be located from the positions of the wavelet into which it is decomposed. This allows the changing spectral composition of non-stationary signals to be measured and compared.

The wavelet transform is defined as the inner product of the signal $x(t)$ with a two-parameter family of basis function

$$WT(b, a) = |a|^{-1/2} \int_{-\infty}^{\infty} x(t)\, \overline{\Psi\left(\frac{t-b}{a}\right)}\, \mathrm{d}t \tag{1}$$

where $\Psi_{b,a} = \Psi((t-b)/a)$ is an oscillatory function, $\bar{\Psi}$ denotes the complex conjugate of Ψ, b is the time delay (translate parameter) which gives the position of the wavelet, and a is the scale factor (dilation parameter) which determines the frequency content. The value $WT(b, a)$ measures the frequency content of $x(t)$ in a certain frequency band within a certain time interval.

To analyze any finite energy signal, the continuous wavelet transform (CWT) uses the dilation and translation of a single wavelet function $\Psi(t)$. The CWT also provides a natural tool for time–frequency signal analysis since each template $\Psi_{b,a}$ is predominantly localized in a certain region of the time–frequency plane with a central frequency that is inversely proportional to a. What distinguishes it from the short-time Fourier transform is the multi-resolution nature of the analysis. However, the CWT is time consuming. From a computational point of view, discrete wavelet transform (DWT) is more efficient and the fast algorithms are available. The DWT algorithm including orthogonal, biorthogonal, semi-orthogonal wavelets, etc., can be found in reference [5]. In general, although the DWT is computationally efficient, the disposition in scale is usually too coarse to track correctly the evolution of the WT coefficients through the scale levels. This only offers the limited detail information. Alternatively, the approach presented in this paper with wavelet filter is a fast algorithm similar to DWT, but it provides detailed signal information around specific frequencies in both the time and frequency domain.

The principal application of the wavelet transform, in analysing the signals, is the determination as well as the description of events localized in time. The time–frequency localization property of the wavelet transform and the existence of fast algorithms make it a tool of choice for the analysis of non-stationary signals, such as speech and image processing, biomedical signal processing, etc.

2.2. WAVELET FILTER DESIGN

A wavelet filter design method based on frequency shift and single wavelet-superposition was introduced in reference [6]. With this method the filters for low-pass, high-pass, band-pass, band elimination, etc., can be designed easily. The pass bandwidth of the filters can be adjusted by changing the parameters of wavelet. For the algorithm presented here, a band-pass filter has been derived to correct errors found in reference [6], and extended the design for a very narrow frequency-band wavelet filter including single central frequency and multiple central frequencies analysis of vibration signals. This has been achieved by using Gaussian wavelets.

Gaussian functions are optimal in terms of their time–frequency localization [7]. The time—frequency localization property of the Gaussian wavelet makes it possible to design filters with very narrow frequency band.

Wavelet $\Psi_{b,a}\,e^{i2\pi f_k t}$ about frequency f_k can be obtained through frequency shift of $\Psi_{b,a}$, then combination wavelets can be constructed by superposing n single wavelets:

$$\Psi_g(t) = \sum_{k=1}^{n} \Psi_{b,a}\, e^{i2\pi f_k t} \qquad (2)$$

Selecting the proper scaling factor a and the frequency centre f_k of the signal wavelet, combination wavelets can be constructed with a plain top frequency window. To form combination wavelets with a plain top frequency window for the Gaussian wavelet, the following condition should be satisfied:

(1) the scaling factor a of all the single wavelets should be the same;
(2) the frequency centre interval Δf between two adjacent single wavelets must be equal;
(3) The Δf should be selected properly.

The Gaussian function is

$$\Psi_a'(t) = \frac{1}{2a\sqrt{\pi a}}\, \mathrm{e}^{-t^2/4a^2\sigma} \quad (\sigma > 0). \tag{3}$$

For $\Psi_a(t) = \mathrm{e}^{-t^2/4a^2\sigma}$, its Fourier transform is

$$\begin{aligned}\hat{\Psi}_a(f) &= \int_{-\infty}^{+\infty} \mathrm{e}^{-\mathrm{i}2\pi ft}\, \mathrm{e}^{-t^2/4a^2\sigma}\, \mathrm{d}t \\ &= \mathrm{e}^{-a^2\sigma(2\pi f)^2}.\end{aligned} \tag{4}$$

It is also a Gaussian function. It has the maximum value when $f = 0$. According to equation (4), the spectrum area of the single Gaussian wavelet is

$$S_1 = \int_{-\infty}^{+\infty} \mathrm{e}^{-a^2\sigma(2\pi f)^2}\, \mathrm{d}f = \frac{1}{2\pi a}\sqrt{\frac{\pi}{\sigma}}. \tag{5}$$

Assuming the combination wavelets (which are constructed by n single Gaussian wavelets) have spectrum amplitude A_g, spectrum area S_g, then $S_g = nS_1 = n\Delta f A_g$, and therefore

$$\Delta f = \frac{1}{2\pi a A_g}\sqrt{\frac{\pi}{\sigma}}. \tag{6}$$

Because the maximum value of the single wavelet spectrum is 1, so $A_g \geqslant 1$, therefore

$$\Delta f_{max} = \frac{1}{2\pi a}\sqrt{\frac{\pi}{\sigma}}. \tag{7}$$

Considering the combination wavelets' spectrum formed by n Gaussian wavelets, the interval between the first and last single wavelet must be larger than Δf_{max} in order to get the top plain spectrum. Thus, the minimum interval between the two single wavelets

$$\Delta f_{min} = \frac{\Delta f_{max}}{(n-1)} = \frac{1}{2\pi a(n-1)}\sqrt{\frac{\pi}{\sigma}}. \tag{8}$$

Equation (8) ensures that the combination wavelets' spectrum has little change in the pass band and is not bell-like.

Therefore, the range of Δf is

$$\frac{1}{2\pi a(n-1)}\sqrt{\frac{\pi}{\sigma}} \leqslant \Delta f \leqslant \frac{1}{2\pi a}\sqrt{\frac{\pi}{\sigma}}. \tag{9}$$

When Δf is selected according to equation (8), the frequency spectrum of combination wavelets is plain, but the frequency window amplitude A_g will change with Δf as

$$A_g = \frac{1}{2\pi a \Delta f}\sqrt{\frac{\pi}{\sigma}}. \tag{10}$$

From equations (9) and (10), combination wavelets with band-pass filter properties can be obtained, which have the form

$$\Psi_g(t) = \mathrm{e}^{-t^2/4a^2\sigma}\,(\mathrm{e}^{\mathrm{i}2\pi f_L t} + \cdots + \mathrm{e}^{\mathrm{i}2\pi(f_L + k\Delta f)t} + \cdots \mathrm{e}^{\mathrm{i}2\pi f_H t})/A_g \quad (k = 0, 1, 2, \ldots, n-1) \tag{11}$$

where f_L and f_H are the central frequency of the first and final wavelet respectively. With properly selected parameters a, σ, Δf, based on equation (11), that equation will have ideal band-pass properties.

The convolution formula of wavelet transform is

$$W(t) = x(t) * \Psi_g(t), \tag{12}$$

where $W(t)$ is the signal with frequency from f_L to f_H of signal $x(t)$.

The scaling factors a, σ are related to the side band property of the filter, while Δf, n can adjust the plain frequency window for band pass. Compared with the FIR filter design method, this method is more efficient, reliable and flexible.

3. SIGNAL PROCESSING METHODS

3.1. SINGLE CENTRAL FREQUENCY ANALYSIS AND MONITORING

The property of time–frequency localization of the Gaussian wavelet has been used for wavelet filter design. The frequency range of the vibration signal is normally $0 \sim 40$ kHz. From equations (8), (10), (11), $a = \sigma = 0{\cdot}0107$, $\Delta f = 1{\cdot}0075$, $n = 9$, $A_g = 7{\cdot}9997$ can be chosen, and thus the frequency error is approximately ± 4 Hz which may be accepted in most cases. Therefore, the very narrow band-pass filter around central frequency f_1 is obtained:

$$\Psi_g(t) = \mathrm{e}^{-204{\cdot}0745t^2}\,(\mathrm{e}^{\mathrm{i}2\pi f_{L_1} t} + \cdots + \mathrm{e}^{\mathrm{i}2\pi f_1 t} + \cdots + \mathrm{e}^{\mathrm{i}2\pi f_{H_1} t})/A_g. \tag{13}$$

Figure 1 is an example of the power spectrum of a wavelet filter around $f_1 = 4746$ Hz. After filtering, the very narrow band signal around the specified f_1 is separated from the vibration signal. Then this signal can be observed in either the time or frequency domain. However, as mentioned above, there exists band-pass frequency error with the filter. The noise in the signal is unavoidable. In order to enhance the signal at central frequency f_1, an autocorrelation algorithm is proposed. The autocorrelation function involves only one signal and provides information about the structure of the signal or its behaviour in the time domain [8].

In this case, the autocorrelation enhancement is for detection and recovery of the signal buried in noise. The autocorrelation enhancement with fast algorithm is defined as follows [9]:

$$R_{xx}(\tau) = F^{-1}\{F_{xx}(\mathbf{x})\}, \tag{14}$$

where

$$F_{xx}(x) = |\hat{X}(f)|^2. \tag{15}$$

$\hat{X}(f)$ is the power spectrum of the signal $x(t)$, and F^{-1} denotes the operation of the inverse transform of the power spectrum. In this paper, it is proposed to use equation (14) for single

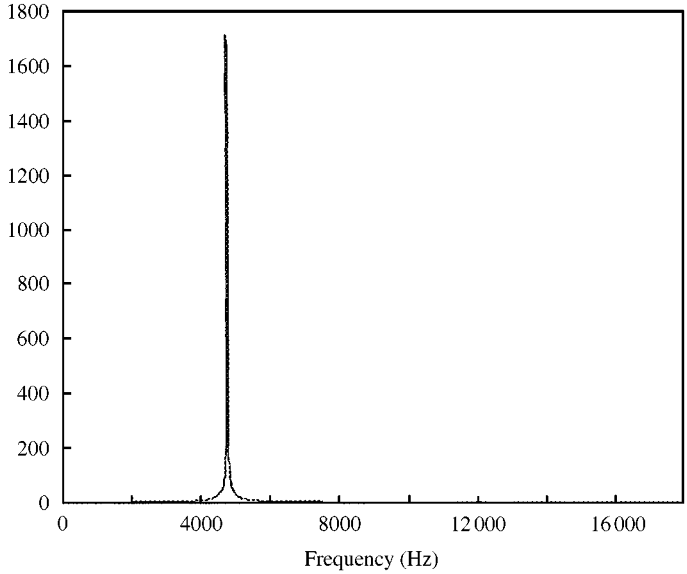


Figure 1. Powder spectrum of a wavelet filter around $f_1 = 4746$ Hz.

central frequency signal enhancement in the time domain, and equation (15) for signal enhancement in frequency domain. With the enhancement, the noise can be reduced effectively. Thus, the change of the signal amplitude at specified frequency with time can be observed more clearly. This provides more detailed information about the signal. Even a small change may be observed. Therefore, the signal is more sensitive to the change of the conditions.

3.2. MULTIPLE CENTRAL FREQUENCIES ANALYSIS AND MONITORING

When extending the wavelet filter design method, a very narrow multi-band-pass filter may also be designed. From equation (13), if m significant and natural frequencies f_1, $f_2, \ldots, f_m$, are specified, then

$$\Psi_g(t) = \mathrm{e}^{-204{\cdot}0745t^2}\,[(\mathrm{e}^{i2\pi f_{L_1}t} + \cdots + \mathrm{e}^{i2\pi f_1 t} + \cdots + \mathrm{e}^{i2\pi f_{H_1}t}) + \mathrm{e}^{i2\pi f_{L_2}t} + \cdots + \mathrm{e}^{i2\pi f_2 t} + \cdots + \mathrm{e}^{i2\pi f_{H_2}t}) + \cdots + \mathrm{e}^{i2\pi f_{L_m}t} + \cdots + \mathrm{e}^{i2\pi f_m t} + \cdots \mathrm{e}^{i2\pi f_{H_m}t})]/A_g. \quad (16)$$

Figure 2 is an example with $m = 5$. With wavelet filtering and enhancement, the signal with specified $f_1, f_2, \ldots, f_m$ is separated from the vibration signal. Then the change in the amplitudes of these frequencies in the frequency domain can be observed at the same time. Therefore, it is convenient to compare the changing amplitudes of all the significant and natural frequencies under different conditions. This provides more reliable and concentrated information. While the signatures corresponding to a certain condition are recognized, it is not difficult to implement a real-time monitoring program, to identify the different conditions and to detect the faults. Since the above signal-processing methods are all with a fast algorithm, it is very suitable for real-time application.

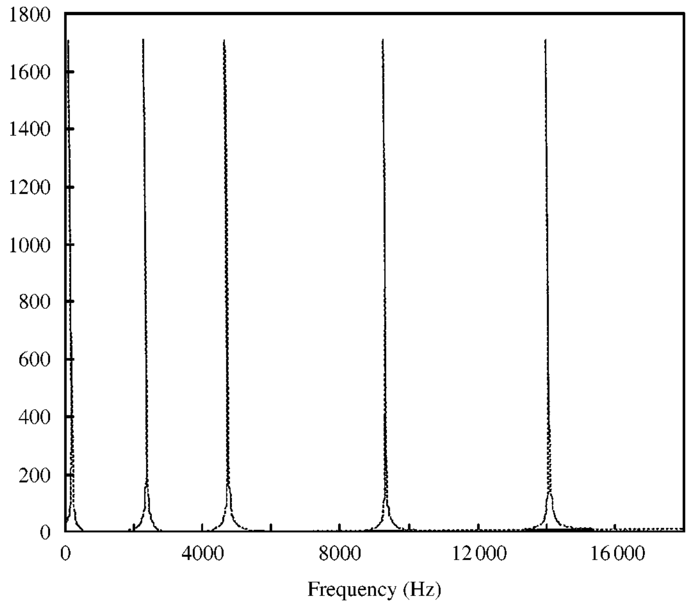


Figure 2. An example of the power spectrum of a wavelet filter around $f_1 = 200$ Hz, $f_2 = 2374$ Hz, $f_3 = 4746$ Hz, $f_4 = 9341$ Hz, $f_5 = 14\,088$ Hz.

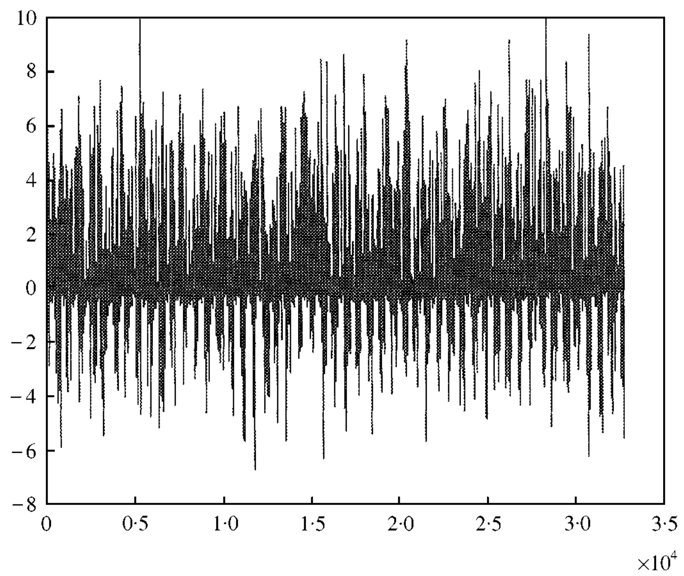


Figure 3. An example of vibration signal.

4. EXAMPLES, RESULTS AND DISCUSSION

Figure 3 is an example of vibration signal measured at a machine spindle nose with free running, and Figure 4 is its power spectrum.

With a wavelet filter, any significant or natural frequency in the time and frequency domain can be observed. Figure 5 is the power spectrum of signal around $f_1 = 4746$ Hz

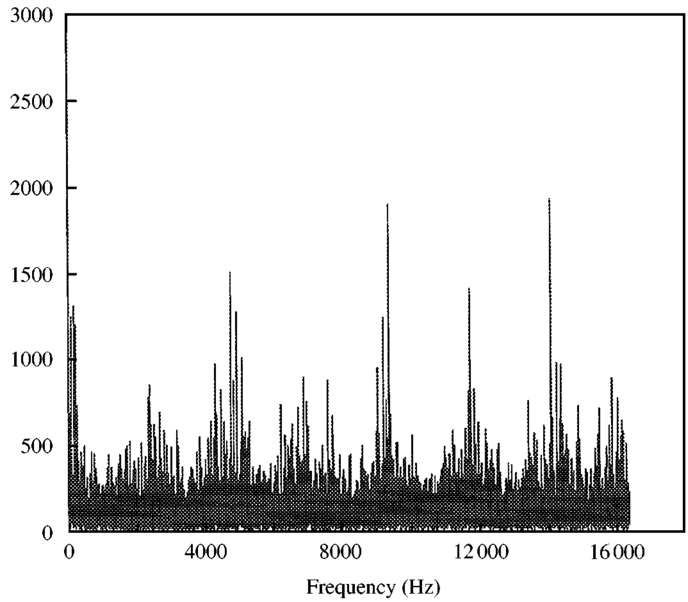


Figure 4. Power spectrum of vibration signal.

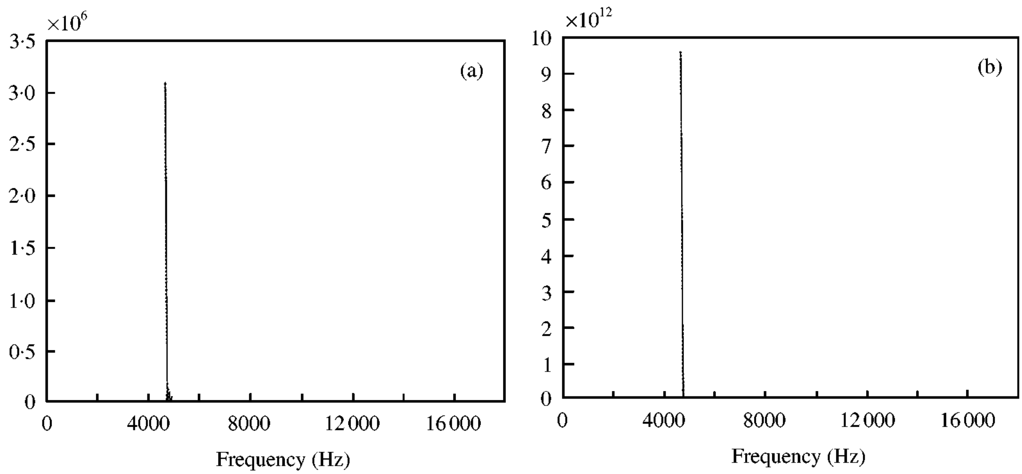


Figure 5. (a) Power spectrum without enhancement, (b) power spectrum with enhancement.

after filtering. Figure 6 is the signal in the time domain. Noise cancellation is achieved with enhancement. If it is necessary to compare the specified multiple central frequencies at the same time, then equation (16) can be used for wavelet filter design. For example, this can be observed for $f_1 = 200$ Hz, $f_2 = 2374$ Hz, $f_3 = 4746$ Hz, $f_4 = 9341$ Hz, and $f_5 = 14\,088$ Hz, Figure 7 is the power spectrum of the signal after filtering. Therefore, the peak ratio defined in reference [4] can be calculated and monitored on line.

Since the peak ratio is the most reliable indicator of localized defect presence, there may exist a relationship between peak value and defect width. Therefore, once the peak ratio verifies that a defect is present, the peak value can then be used to estimate the size of defect

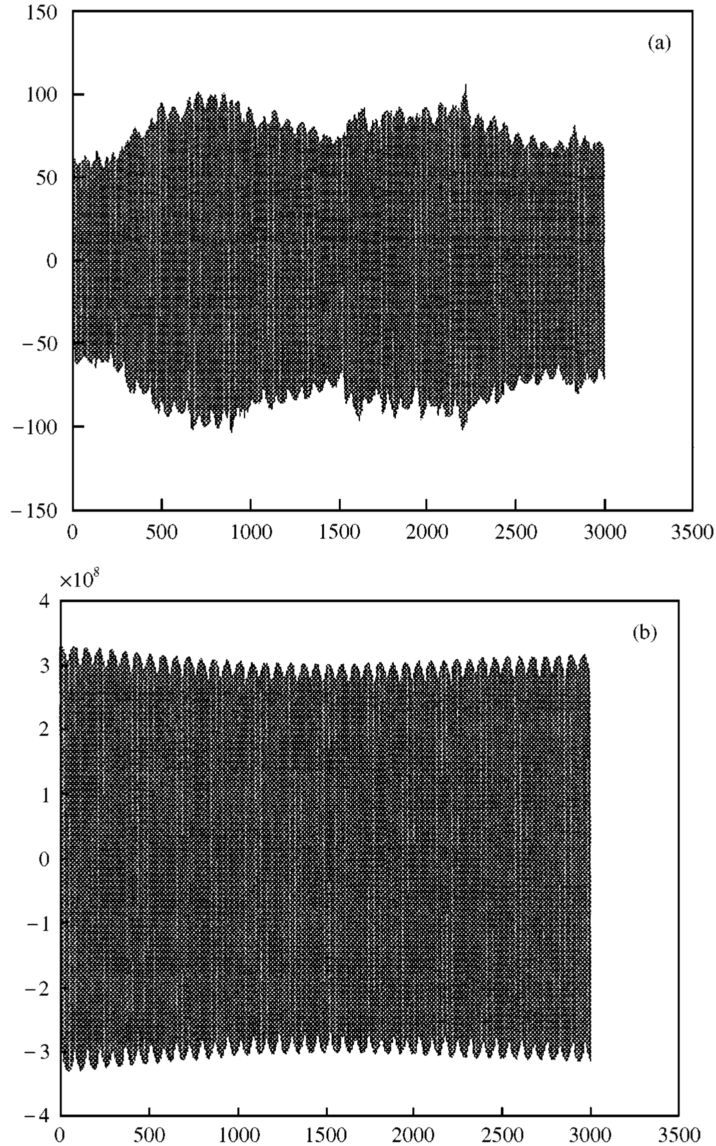


Figure 6. (a) Signal in the time domain without enhancement, (b) signal in the time domain with enhancement.

[4]. Thus it may be used for condition and defect monitoring. On the other hand , the signal with a specified frequency in the time domain, which provides more detailed information, is another condition indicator. The advantage of observing the signal in the time domain is that any small change in condition may possibly result in the change of amplitude with time. It is more sensitive to the condition. However, the approach presented as here is based on the correct selection of significant and natural frequencies, testing and a great deal of experimental data are needed to establish a correlation between the condition status and the signal feature for a certain machine. Then the in-process condition monitoring can be implemented with this approach with a fast algorithm.

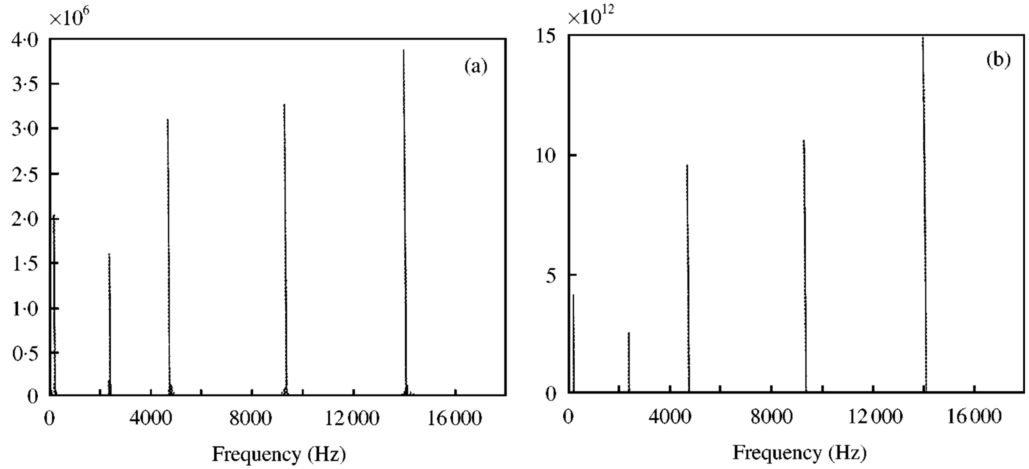


Figure 7. (a) Power spectrum without enhancement, (b) power spectrum with enhancement.

Condition monitoring refers to the classification of the data based on the fundamental understanding of the condition status so that an abnormal situation can be detected and fault modes isolated. However, the sensitivity of the signal feature to classifying non-normal conditions is dependent on the data used. Timely and correct interpretation of processed data will lead to improved quality, safer operation and waste reduction. Although vibration signals carry a great deal of information about a system condition, without proper pre-treatment, the necessary interpretation is difficult, and even impossible. Gross data must be processed or modified and de-noised. In order to generate useful insights into the problem of condition monitoring, the wavelet filter with autocorrelation enhancement has been designed. The condition monitoring activity is then carried out in the time and frequency domain with specified significant and natural frequencies.

5. FAULT DETECTION OF GEARBOX IN HELICOPTER

Applications of wavelet transform to vibration signals for fault detection have been proposed recently. Wang *et al.* [10] used discrete orthogonal wavelets for vibration transient analysis and early gear damage detection. Wang *et al.* [11] also used discrete non-orthogonal wavelets for vibration transient representation and fault detection in a gearbox, with the possibility of narrowing the frequency band in finer scale at the cost of increased computing time. A commonly used technique named time domain averaging has been applied in these papers, which uses a signal synchronous with the shaft rotation to produce the time history of a particular gear. However, these methods of signal processing with wavelet transform have either low resolution of features in the frequency-band scale for detailed analysis or are very time-consuming.

Using the wavelet filter developed here, the original vibration signals can be analyzed without any additional technique such as time domain averaging (which can save computing time), and makes it possible to identify the time when a fault occurs.

An example of a vibration signal used for the algorithm was recorded from CH-46E helicopter. The original vibration signal is shown in Figure 8 and 9 is the power spectrum (frequency domain). In this example, only one faulted component (planetary bearing corrosion) was present during the data collection. The sampling frequency is

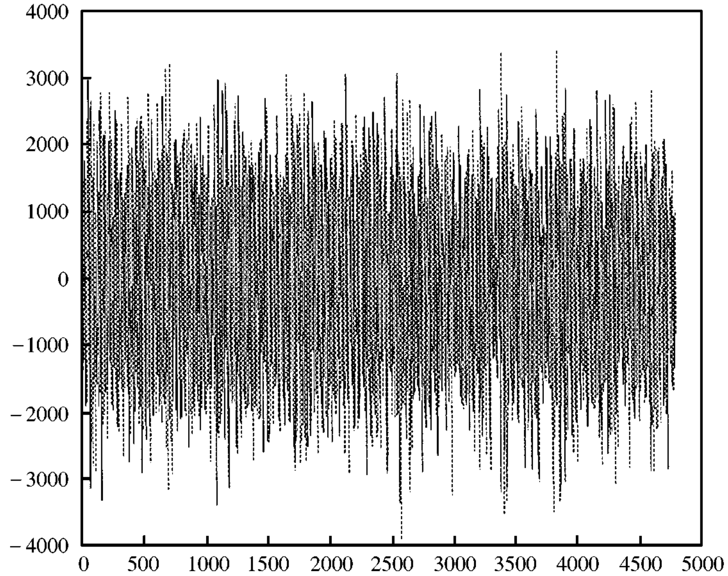


Figure 8. Vibration signal from helicopter.

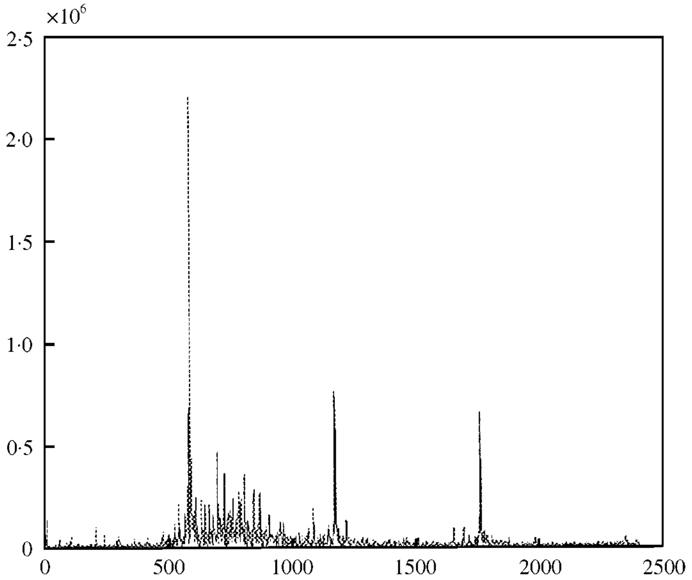


Figure 9. Vibration signal in the frequency domain.

$f_s = 25779{\cdot}01$ Hz. The significant frequencies specified are shown in Figure 9 to be around $f_1 = 3155$ Hz, $f_2 = 6300$ Hz, $f_3 = 9456$ Hz.

5.1. MULTIPLE CENTRAL FREQUENCIES ANALYSIS AND MONITORING

The signal length is $L = 4800$. The signal is separated into six segments with each segment of 800 points. The filter with the specified central frequencies in the frequency

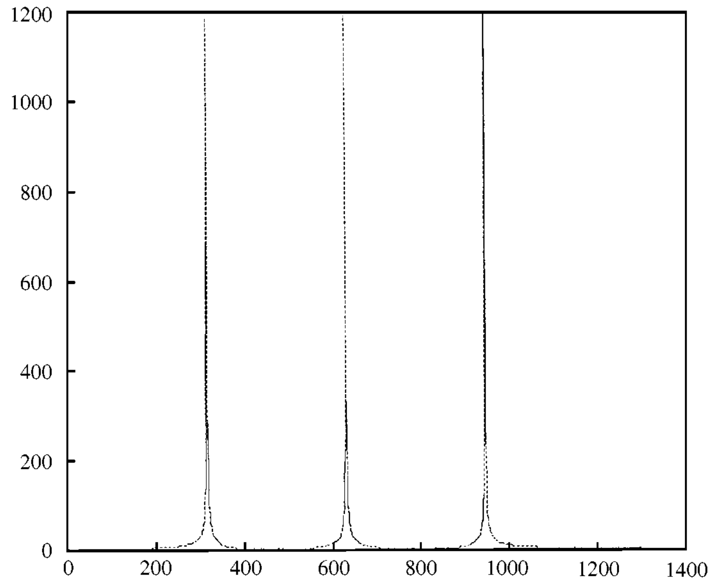


Figure 10. Filter in the frequency domain.

domain is shown in Figure 10. Figure 11(a)–(f) show the signals in the frequency domain corresponding to segments 1–6 respectively. The amplitudes in Figure 11(a) (1–800 points), Figure 11(b) (801–1600 point) and Figure 11(c) (1601–2400 points) are quite similar. This shows that from 1 to 2400 points, there is no great change of machine condition, which means that the gearbox is under normal operation. However, from Figures 11(d) (2401–3200 points) to Figure 11(f) (4001–4800 points), there is significant change in the three peaks. The peak of f_1 goes down and the peaks of f_2 and f_3 rise. This indicates a change in the machine condition between 2401 and 3200 points, i.e., a possible fault occurring (in this example there was a known fault condition). This experiment demonstrates that the peak ratio of significant and natural frequencies in the frequency domain is also a good indicator of condition.

5.2. SINGLE CENTRAL FREQUENCY ANALYSIS AND MONITORING

Multiple central frequencies analysis is based on the frequency domain information. For a single central frequency, the signal information in the time domain can be achieved with wavelet filter. Figure 12(a)–(c) show the time domain information around f_1, f_2, f_3 respectively. From these Figures, it can be seen that the amplitudes start to change significantly around the 2480 point, which may be the time when the fault starts. This provides the same results as above but gives more detailed information with its evolution in the time domain.

By comparing these two methods, it can be seen that both provide reliable indication of the machine condition, and can be used for fault detection. The peak values of multiple central frequencies in the frequency domain can indicate the size of defect, while single central frequency in the time domain is very sensitive to the condition, and can provide accurate time information on the condition.

With wavelet filter by means of significant and natural frequencies analysis, the machine condition can be monitored in real time. To carry out the analysis the algorithms have been implemented using a MMX Pentium 133 MHz PC workstation with a 32 MB RAM memory. This performed the fault detection for the vibration signal from gearbox shown in Figure 8. The computational time was difficult to measure due to the speed of the algorithm. However, a time of approximately 0·8 s for single-frequency analysis (4800 points) and 0·2 s for multiple frequencies analysis (800 points) was estimated.

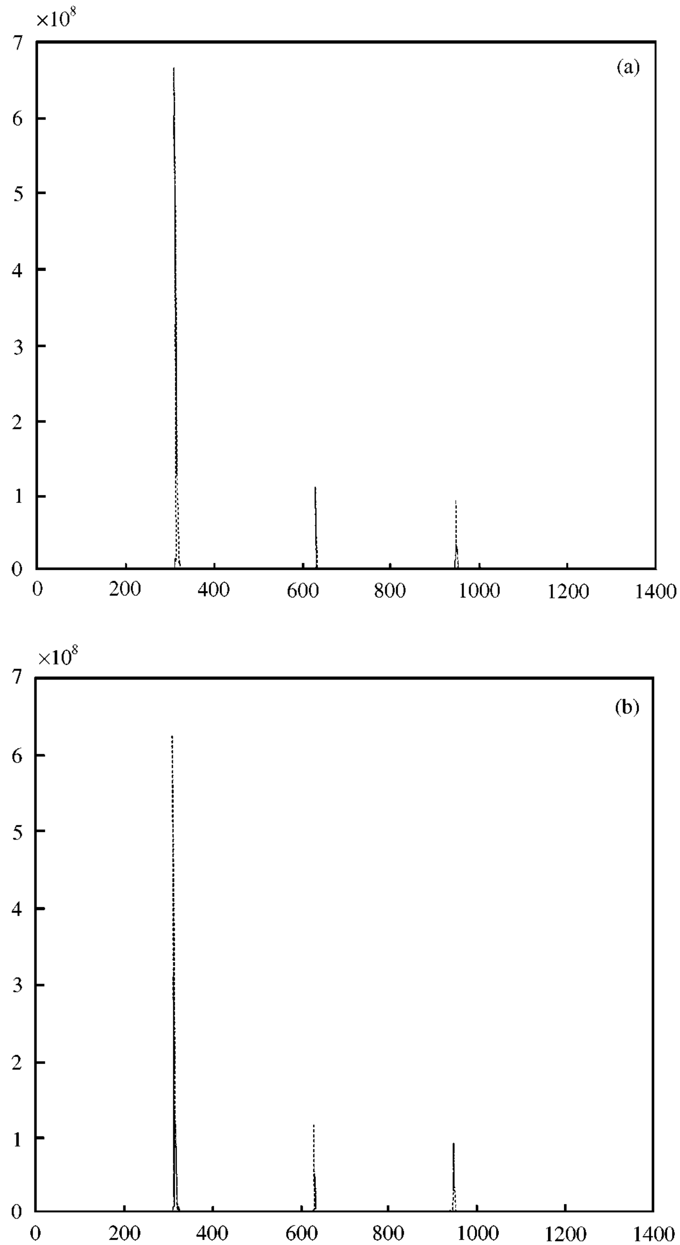


Figure 11. (a) Signal (1–800 points) in the frequency domain, (b) signal (801–1600 points) in the frequency domain, (c) signal (1601–2400 points) in the frequency domain, (d) signal (2401–3200 points) in the frequency domain, (e) signal (3201–4000 points) in the frequency domain, (f) signal (4001–4800 points) in the frequency domain.

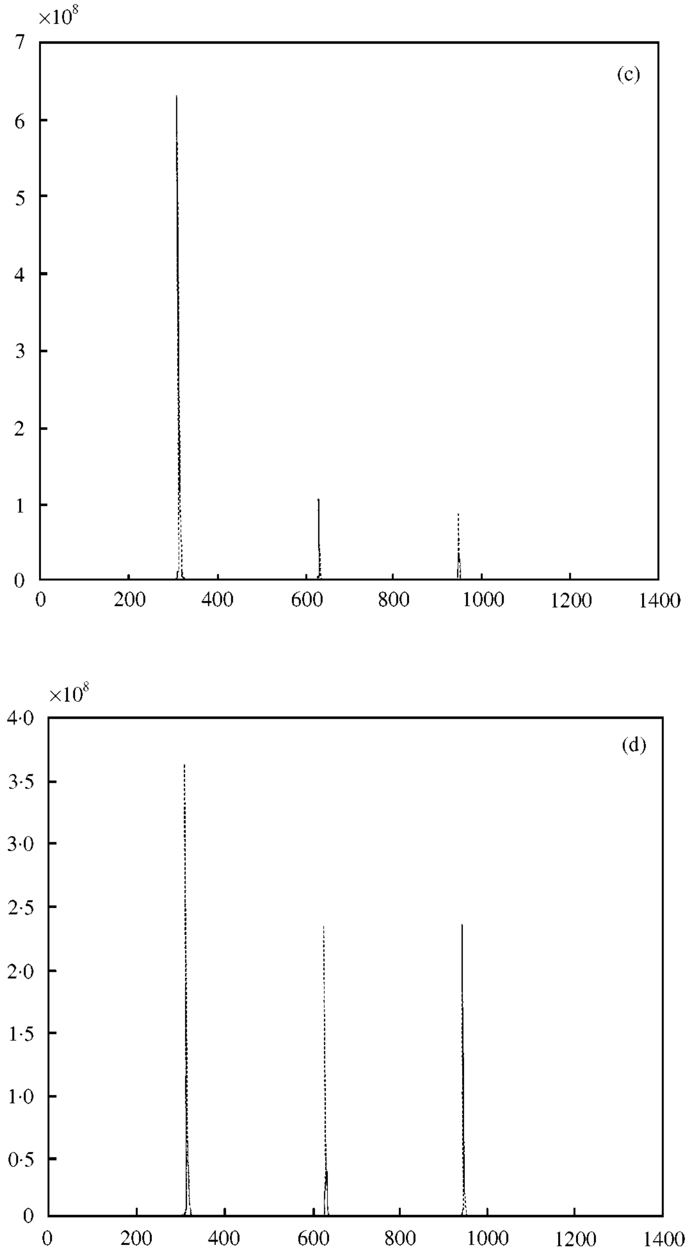
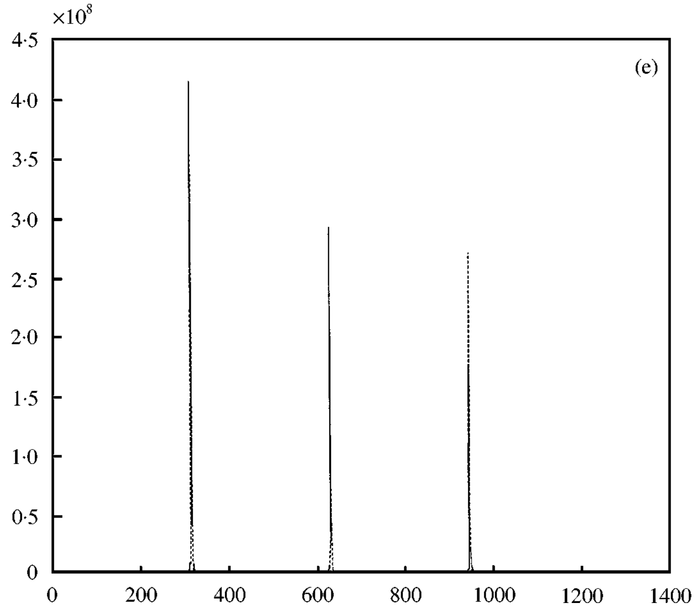


Figure 11. Continued

6. CONCLUSIONS

A condition monitoring approach via vibration signal analysis with wavelet filter design based on the time–frequency localization of the wavelet transform is proposed. Together with autocorrelation enhancement, any number of natural frequencies in either the time or the frequency domain can be observed. The calculated peak ratio and peak value in the



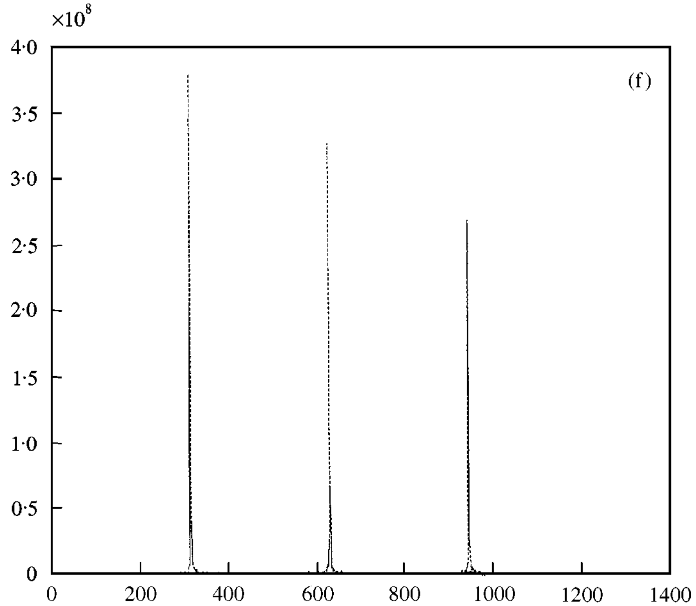


Figure 11. Continued

frequency domain is an indicator of condition and defect, while the detailed information of the vibration signal with a specified frequency is the record of evolution in amplitudes. This enables the detection of a fault at an early stage and the monitoring of its evolution on a time scale. Thus, the important and useful information buried in the vibration signals is revealed. Furthermore, the fast algorithms make it very suitable for real-time monitoring.

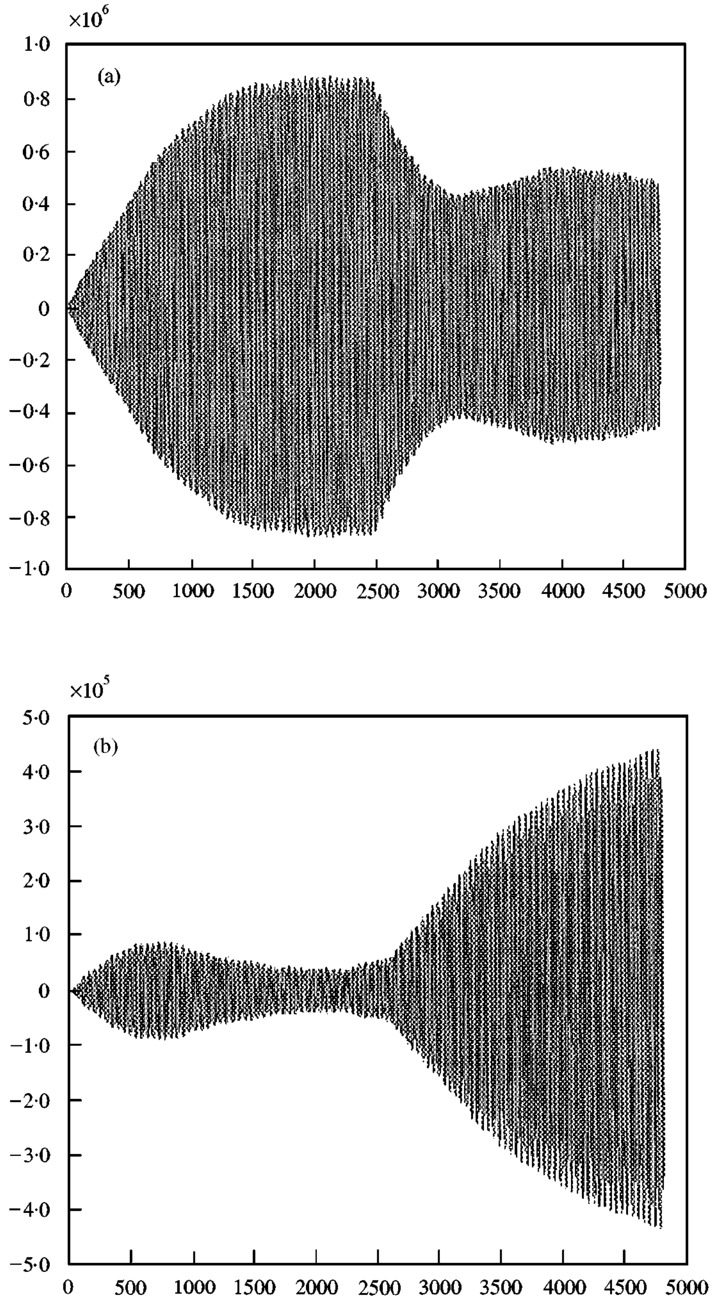


Figure 12. (a) Signal around f_1 in the time domain, (b) signal around f_2 in the time domain, (c) signal around f_3 in the time domain.

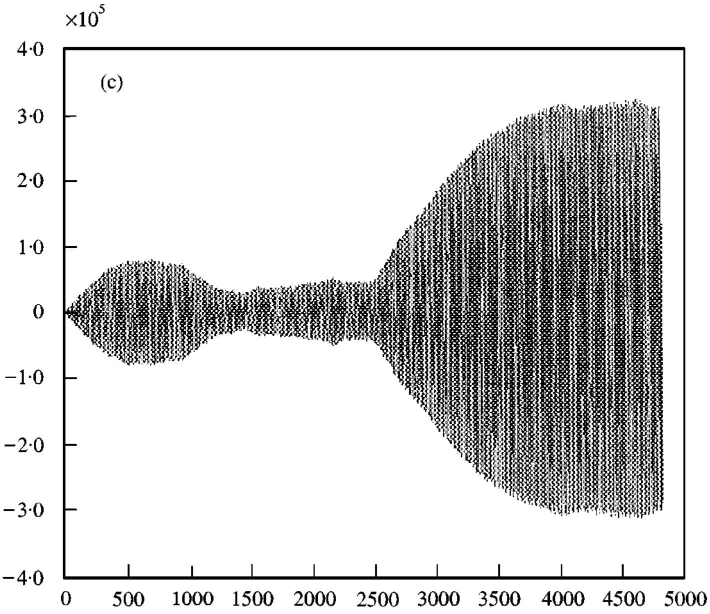


Figure 12. Continued

ACKNOWLEDGMENT

The authors wish to acknowledge Dr Homer Rahnejat and his team in the Department of Mechanical and Manufacturing Engineering, University of Bradford, for their assistance. The authors would also like to thank Dr P. D. McFadden, Department of Engineering Science, University of Oxford, for his help.

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